151 Parts to Infinity

Danko Arlington and an innovative team cast a mathematical monument. **BY DENISE KAPEL, SENIOR EDITOR**

Dr. Ferguson examines his Umbilic Torus.

This CAD drawing represents the Deltoid Torus design.

For more than two years, Dr. Helaman Ferguson, PhD, a mathematician who specializes in stone and bronze sculptures, worked with Danko Arlington Inc., a Baltimore job shop specializing in aluminum and bronze sand castings, to create a towering sculpture with infinite lines. The castings for the outer shape of the piece were produced without pattern tooling, and the final assembly weighs more than 20,000 lbs. The total weight of the sculpture is approximately 65 tons.

Philanthropists James and Marilyn Simons commissioned Ferguson to create the outdoor centerpiece for the new Simons Center for Geometry and Physics at Sony Brook University, Long Island, N.Y. According to John Morgan, founding director, the goal of the center is to bring together mathematicians—in particular, geometers and theoretical physicists—to inform and learn from each other, and to work on problems of common interest with the long-range goal of better understanding connections that, once understood, will transform each subject.

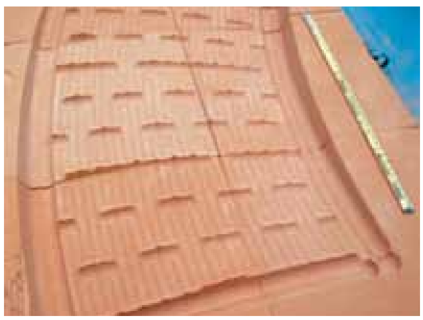
"For the team that constructed the Umbilic Torus SC, the implications of a project benefactor who values the combination of the words 'renaissance' and 'technologies' is no mere coincidence," said John Danko, president, Danko Arlington.

Renaissance Technologies LLC, East Setauket, N.Y., is James Simons' investment management company. Noting the inspiration of the Renaissance following the Dark Ages, Danko cited patrons such as the Medici family of Florence, Italy, who funded innovations in art, science and technology from Lorenzo Ghiberti, Filippo Brunelleschi, Leonardo da Vinci and Michelangelo Buonarrati.

"Today, we would [say] 'thinking outside the box' best summarizes their kind of creativity," Danko said. "So, too, does the design and fabrication of this infinitely curved cast artwork symbolize a new way of thinking." He said the one-off assembly of castings was unique not only due to its size and shape but for the ingenuity employed in its creation and design.

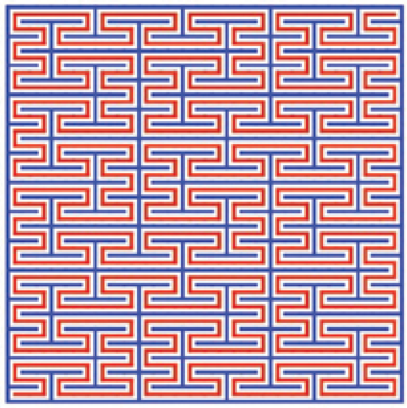
A Custom Collaboration

"Countless hours of collaboration were spent between Dr. Ferguson and his team of foundrymen, architects, planners, structural engineers, stone masons, and representatives from the university and the Simons Foundation," Danko said.



Each panel was 5-axis CNC machined into a 2,400-lb. nobake sand drag.

One hundred and forty-four sections were cast to create the hollow torus shape. The sculpture surface involved a complex equation called a 3-adic curve with lines that, when wrapped around the torus, have no end and no beginning.



Danko Arlington specializes in military and commercial aluminum and bronze sand castings. The company operates a pattern shop, foundry and CNC machine shop in one facility and offers plating, painting, inserts and light assembly. In operation since 1920, it expanded its operations in 2008, overtaking two adjacent buildings and installing an efficient system for reclaiming 100% of its spent foundry sand. It recently became an SBA Certified HubZone Concern, providing job training, steady employment and an opportunity for local residents to rebuild their lives.

This two-year project included casting two smaller torus casting assemblies of one tenth scale and quarter scale. Danko Arlington cast the Umbilic Torus SC (Simons Center) sculpture entirely out of silicon bronze, and the finished piece was welded together as an assembly at neighboring New Arts Foundry, a Baltimore shop that specializes in investment casting bronze.

The 144 sections creating the hollow torus shape were sand cast individually in nobake molds, as was an internal inverted sand cast cap for mounting and six internal buttresses for additional support.

The 2,400-lb. drag section of the mold for each unique, twisted torus panel with its associated 3-adic surface curve was five-axis CNC machined. Ferguson used Mathematica technical computing software from Wolfram, Champaign, Ill., to create each torus section, and he performed hand calculations to program the 3-adic surface filling curve.

"No off-the-shelf CAD/CAM system exists for doing the continuous 3-adic surface filling curve tool path definitions, let alone with all the (normal bundle) angles associated with the surface," said Danko. "Only a PhD of mathematics could crunch so many complex geometries, as this task was truly mind blowing to create in such a scale." The G-code generated Mathematica movements were compiled and downloaded to an XR4050 five-axis gantry machine tool. "This was an older, used X-Y-Z gantry milling machine [Ferguson] retrofitted with two more axis movements to make it completely five-axis compatible," he said.

The advantage with this gantry machine, according to Danko, is the X and Y pathways' elevation above the mold, which keeps the abrasive sand away from the travels. In addition, the Z arm is retrofitted with two more movements. "So, the tool was able to have yaw, W, and pitch, B," he said. "The controllers for the yaw and pitch were also safely out of the way of sand flying from the diamond cutter."

Thousands of tool path movements accurately made the cavity for each section, which was serialized for its shape and orientation so it could be matched to its mating piece later, during fabrication.

Mathematical Magic

The piece is based on a set of parametric equations that create a deltoid, a type of hypocycloid which looks like a triangle with concave sides. The shape is created by rolling a fixed point on a circle inside another, larger circle.

"Anyone who had a Spirograph drawing set most likely made this design," Danko said. It is represented in mathematics as:

$$x(\theta) = (R - r)\cos\theta + r\cos\left(\frac{R - r}{r}\theta\right)$$

$$y(\theta) = (R - r)\sin\theta - r\sin\left(\frac{R - r}{r}\theta\right)$$

When the deltoid is revolved 360 degrees around a perpendicular Z axis while simultaneously twisting 120 degrees, the result becomes a single edge twisted dough-nut-shaped solid called a deltoid torus. The equations are as follows:

$$x = \sin u \left(7 + \cos\left(\frac{u}{3} - 2v\right) + 2\cos\left(\frac{u}{3} + v\right)\right)$$

$$y = \cos u \left(7 + \cos\left(\frac{u}{3} - 2v\right) + 2\cos\left(\frac{u}{3} + v\right)\right)$$

$$z = \sin\left(\frac{u}{3} - 2v\right) + 2\sin\left(\frac{u}{3} + v\right)$$

$$\text{for } -\pi \leq u \leq \pi, \quad -\pi \leq v \leq \pi$$

In addition to the infinite edge the shape develops on the torus, this sculpture surface contains another complex equation called a 3-adic curve.

"The 3-adic lines are akin to ancient Greek or Mayan perpendicular-lined patterns which, when wrapped around the torus, have no end and no beginning," Danko explained.

Obstacles Overcome

Initially, the team encountered a challenge when cutting the molds. Hand ramming each sand block created inconsistencies in its density.

"Areas that were too compact placed more stress on the cutter, which created more heat and friction on the tool. This caused the system to overload and often shut down," said Danko. Experimentation and feedback resulted in improved hand ramming and more uniform density blocks for the process, so the movements flowed more uniformly. The sand was vacuumed by hand.

Each drag, known to Ferguson's art team as a "sand stone," was then hand clayed following each 3-adic curve for the approximate 3/8-inch wall thickness to make its matching cope.

"In this way, the need for a machined sand cope was eliminated, saving time, cost and logistics," said Danko. "[Each] cope was molded right on top of each custom drag mold."

The clay was removed and the drag was hand cut for risering and gating. Once assembled, Danko Arlington filled the mold with C87300 silicon bronze (Everdur).

A stress analysis resulted in the recommendation to add support to the inverted cap inside the torus assembly, to better distribute its weight.

The pattern for each buttress was hand fitted, and these stiffeners as well as the cap were cast from the same alloy as the sculpture to ensure consistent welds and joints.

A bronze cap casting for mounting was produced using a traditionally made hardwood pattern with a machined foam corebox.

"Cutting polystyrene panels and hand-fitting inside the sculpture was easy to manage, compared to heavy bronze sheets," Danko said. "Once fit to size with a knife and scissors, it was sent to the foundry to ram up as a pattern for sand casting."

The large bronze cap casting inside the sculpture was produced from a traditionally made hardwood pattern with a machined foam corebox. The cap was machined at Danko Arlington and included a variety of tapped holes for locking bolts and leveling screws.

Assembling Infinity

New Arts Foundry craftsmen straightened and welded nine cast panel pieces and assembled individual 22.5-degree bulkhead sections on top of each other, piece by piece, a process Danko likens to shipbuilding. Eight bulkheads were welded together to produce a half torus for transport to the university before final, permanent assembly onsite. The entire sculpture was placed on a stainless steel post, which was part of the foundation for the sculpture. Each section joint and 3-adic curve was textured to match the as-cast surface.

Eight bulkheads were welded together to produce a half torus for transport to final assembly onsite.

"The silicon bronze was finished in a light green verde patina, which will take on its own color as it weathers," said Danko.

The sculpture is surrounded by a base consisting of nine slabs of Lake Superior granite built in the shape of a circle and deltoid section, engraved with the associated equations. Dedicated on October 25, 2012, Ferguson's Umbilic Torus SC sand cast sculpture epitomizes the lifetime achievement of an innovative mathemetician and sculptor. MC